AH National Qualifications
EXEMPLAR PAPER ONLY

EP24/AH/01 Mathematics

Date — Not applicable

Duration — 3 hours

Total marks — 100

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet, you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

EP24AH01

FORMULAE LIST

Standard derivatives	
$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax)+c$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
e^{ax}	$\frac{1}{a}e^{ax}+c$

Summations

(Arithmetic series) $$S_n = \frac{1}{2}n\left[2a+(n-1)d\right]$$

(Geometric series) $$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}, \quad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r \quad \text{where} \quad \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Maclaurin expansion

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \ldots$$

FORMULAE LIST (continued)

De Moivre's theorem

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

Vector product

$$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Matrix transformation

Anti-clockwise rotation through an angle, θ, about the origin, $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

MARKS

Total marks — 100

Attempt ALL questions

1. Use the binomial theorem to expand and simplify

$$\left(\frac{x^2}{3}-\frac{2}{x}\right)^5.$$ 4

2. Given $y=e^{\cos x}\sin^2 x$

(a) Find $\frac{dy}{dx}$. 3

Given $f(x)=\frac{x^2-1}{x^2+1}$

(b) Obtain $f'(x)$ and simplify your answer. 3

3. Use Gaussian elimination on the system of equations below to give an expression for z in terms of λ.

$$x+y+z=2$$
$$4x+3y-\lambda z=4$$
$$5x+6y+8z=11$$

Determine the value(s) of λ for which this system **does not** have a solution. 5

4. The velocity, v, of a particle, P, at time, t, is given by

$v=e^{3t}+2e^t$.

(a) Find the acceleration of P at time t. 2

(b) Find the distance covered by P between $t=0$ and $t=\ln 3$. 2

5. Given that $z=1-\sqrt{3}i$, write down the conjugate $\bar{z}$ and express $\bar{z}^2$ in polar form. 4

6. The equation $x^4+y^4+9x-6y=14$ defines a curve passing through the point A$(1,2)$.

Find the equation of the tangent to the curve at A. 4

MARKS

7. Matrices A and B are defined by $A=\begin{pmatrix} 4 & p \\ -2 & 1 \end{pmatrix}$ and $B=\begin{pmatrix} x & -6 \\ 1 & 3 \end{pmatrix}$.

(a) Find A^2. **1**

(b) Find the value of p for which A^2 is singular. **2**

(c) Find the values of p and x if $B=3A'$. **2**

8 (a) Give the first three non-zero terms of the Maclaurin series for $\cos 3x$. **2**

(b) Write down the first four terms of the Maclaurin series for e^{2x}. **1**

(c) Hence, or otherwise, determine the Maclaurin series for $e^{2x}\cos 3x$ up to, and including, the term in x^3. **3**

9. Prove by contradiction, that if x is irrational then $\sqrt{x}$ is irrational. **3**

10. Find the coordinates of the point of inflexion on the graph of $y=\sin x+\tan x$, where $-\frac{\pi}{2}<x<\frac{\pi}{2}$. **5**

11. (a) Write down the 2 × 2 matrix, M_1, associated with a reflection in the y-axis. **1**

(b) Write down a second 2 × 2 matrix, M_2, associated with an anti-clockwise rotation through an angle of $\frac{\pi}{2}$ radians about the origin. **1**

(c) Find the 2 × 2 matrix, M_3, associated with an anti-clockwise rotation through $\frac{\pi}{2}$ radians about the origin followed by a reflection in the y-axis. **1**

(d) State the single transformation associated with M_3. **1**

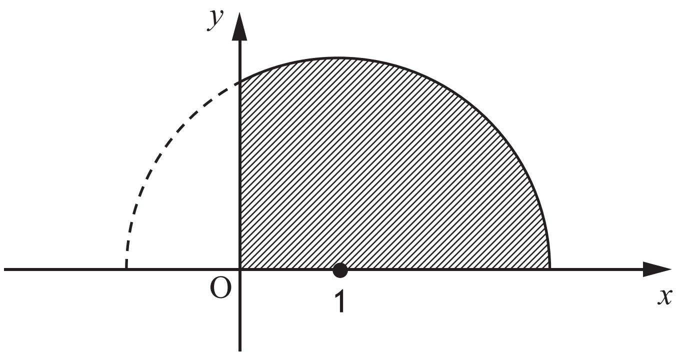
12. Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n}\frac{1}{r(r+1)}=1-\frac{1}{n+1}.$$ **5**

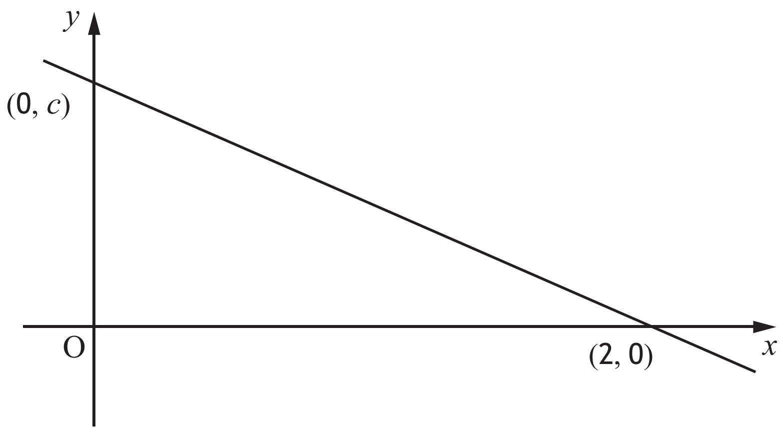
MARKS

13. A semi-circle with centre $(1, 0)$ and radius 2, lies on the x-axis as shown.

Find the volume of the solid of revolution formed when the shaded region is rotated completely about the x-axis. **5**



14. Part of the straight line graph of a function $f(x)$ is shown.



(a) Sketch the graph of $f^{-1}(x)$ showing points of intersection with the axes. **2**

(b) State the value of k for which $f(x)+k$ is an odd function. **1**

(c) Find the value of h for which $|f(x+h)|$ is an even function. **2**

15. Use the substitution $x = \tan\theta$ to determine the exact value of

$$\int_0^1 \frac{dx}{\left(1+x^2\right)^{\frac{3}{2}}}.$$ **5**

16. A line, L_1, passes through the point P(2, 4, 1) and is parallel to

$$\mathbf{u}_1 = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$$

and a second line, L_2, passes through Q(−5, 2, 5) and is parallel to

$$\mathbf{u}_2 = -4\mathbf{i} + 4\mathbf{j} + \mathbf{k}\,.$$

(a) Determine the vector equations for L_1 and L_2. **2**

(b) Show that the lines L_1 and L_2 intersect and find the point of intersection. **4**

(c) Determine the equation of the plane containing L_1 and L_2. **3**

17. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x + 12\,.$$ **7**

(b) Find the particular solution for which $y = -\frac{3}{2}$ and $\frac{dy}{dx} = \frac{1}{2}$ when $x = 0$. **3**

18. Vegetation can be irrigated by putting a small hole in the bottom of a cylindrical tank, so that the water leaks out slowly.

Torricelli's law states that the rate of change of volume, V, of water in the tank is proportional to the square root of the height, h, of the water above the hole.

This is given by the differential equation:

$$\frac{dV}{dt} = -k\sqrt{h} \quad ,k > 0.$$

(a) For a cylindrical tank with constant cross-sectional area, A, show that the rate of change of the height of the water in the tank is given by

$$\frac{dh}{dt} = \frac{-k}{A}\sqrt{h}.$$ **2**

(b) Initially, when the height of the water is 144 cm, the rate at which the height is changing is −0·3 cm/hr.

By solving the differential equation in part (a), show that

$$h = \left(12 - \frac{1}{80}t\right)^2.$$ **4**

(c) How many days will it take for the tank to empty? **2**

(d) Given that the tank has radius of 20 cm, find the rate, in cm^3/hr, at which the vegetation was receiving water from the tank at the end of the fourth day. **3**

[END OF EXEMPLAR QUESTION PAPER]