AH National Qualifications
EXEMPLAR PAPER ONLY

EP24/AH/01 **Mathematics**

Marking Instructions

These Marking Instructions have been provided to show how SQA would mark this Exemplar Question Paper.

General Marking Principles for Advanced Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the Detailed Marking Instructions, which identify the key features required in candidate responses.

(a) Marks for each candidate response must <u>always</u> be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(d) Working subsequent to an error must be followed through, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is easier, candidates lose the opportunity to gain credit.

(e) Where transcription errors occur, candidates would normally lose the opportunity to gain a processing mark.

(f) Scored-out or erased working which has not been replaced should be marked where still legible. However, if the scored-out or erased working has been replaced, only the work which has not been scored out should be judged.

(g) Unless specifically mentioned in the Detailed Marking Instructions, do not penalise:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in solutions
- repeated errors within a question

Definitions of Mathematics-specific command words used in this Exemplar Question Paper

Determine: determine an answer from given facts, figures, or information.

Expand: multiply out an algebraic expression by making use of the distributive law or a compound trigonometric expression by making use of one of the addition formulae for $\sin(A \pm B)$ or $\cos(A \pm B)$.

Express: use given information to rewrite an expression in a specified form.

Find: obtain an answer showing relevant stages of working.

Hence: use the previous answer to proceed.

Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used.

Prove: use a sequence of logical steps to obtain a given result in a formal way.

Show that: use mathematics to show that a statement or result is correct (without the formality of proof) — all steps, including the required conclusion, must be shown.

Sketch: give a general idea of the required shape or relationship and annotate with all relevant points and features.

Solve: obtain the answer(s) using algebraic and/or numerical and/or graphical methods.

Detailed Marking Instructions for each question

Question			Expected response (Give one mark for each •)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each •)
1			Ans: $\frac{x^{10}}{243}-\frac{10x^7}{81}+\frac{40x^4}{27}-\frac{80x}{9}+\frac{80}{3x^2}-\frac{32}{x^5}$	4	
			•1 correct unsimplified expansion		•1 $= {}^5C_0\left(\frac{x^2}{3}\right)^5 + {}^5C_1\left(\frac{x^2}{3}\right)^4\left(\frac{-2}{x}\right)^1 + {}^5C_2\left(\frac{x^2}{3}\right)^3\left(\frac{-2}{x}\right)^2 + {}^5C_3\left(\frac{x^2}{3}\right)^2\left(\frac{-2}{x}\right)^3 + {}^5C_4\left(\frac{x^2}{3}\right)\left(\frac{-2}{x}\right)^4 + {}^5C_5\left(\frac{-2}{x}\right)^5$
			•2 fully simplified powers of x		•2, 3, 4 $=\frac{x^{10}}{243}-\frac{10x^7}{81}+\frac{40x^4}{27}-\frac{80x}{9}+\frac{80}{3x^2}-\frac{32}{x^5}$
			•3 powers of 3 and binomial coefficients **OR** powers of -2 correct		
			•4 complete and simplifies correctly		
Notes: 1.1 Accept negative powers of x . 1.2 Coefficients must be fully processed to simplified fractions and whole numbers.					
2	a		Ans: $e^{\cos x}.2\sin x\cos x+\sin^2 x.e^{\cos x}(-\sin x)$	3	
			•1 evidence of product rule		•1 see 2.1
			•2 first term		•2 $e^{\cos x}.2\sin x\cos x+...$
			•3 second term		•3 $...+\sin^2 x.e^{\cos x}(-\sin x)$
2	b		Ans: $f'(x)=\frac{4x}{\left(x^2+1\right)^2}$	3	
			•4 know to use quotient (or product) rule		•4 $f'(x)=\frac{(x^2+1).2x-(x^2-1).2x}{(x^2+1)^2}$
			•5 correct derivative, using either rule, unsimplified		•5 $=\frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}$

			•^{6} simplify answer		•^{6} $=\dfrac{4x}{\left(x^2+1\right)^2}$
					OR
			•^{4} use polynomial division (or inspection) correctly simplify $f(x)$		•^{4} $f(x)=1-\dfrac{2}{x^2+1}$
			•^{5} correctly complete first step		•^{5} $f'(x)=-1(-2)(x^2+1)^{-2}\ldots$ $\times(2x)$
			•^{6} apply chain rule and simplify answer		•^{6} $\therefore f'(x)=4x\left(x^2+1\right)^{-2}$ $=\dfrac{4x}{\left(x^2+1\right)^2}$
Notes: 2.1 Evidence of method: statement of the rule and evidence of progress in applying it. OR Application showing the **sum** of two terms, both involving differentiation. 2.2 Sign switched •^{1} •^{3} available for $e^{\cos x}\sin^3 x-e^{\cos x}2\sin x\cos x$ or equivalent. 2.4 Evidence of method: statement of the rule and evidence of progress in applying it. OR Application showing the ***difference*** of two terms, both involving differentiation and a denominator. 2.5 Accept use of product rule with equivalent criteria for •^{4}.					
3			Ans: $\lambda=-1$	5	
			•^{1} set up augmented matrix		•^{1} $\left.\begin{array}{ccc\|c} 1 & 1 & 1 & 2 \\ 4 & 3 & -\lambda & 4 \\ 5 & 6 & 8 & 11 \end{array}\right.$
			•^{2} obtain zeroes in first elements of second and third rows		•^{2} $\left.\begin{array}{ccc\|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 4+\lambda & 4 \\ 0 & 1 & 3 & 1 \end{array}\right.$
			•^{3} complete elimination to upper triangular form		•^{3} $\left.\begin{array}{ccc\|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 4+\lambda & 4 \\ 0 & 0 & 1+\lambda & 3 \end{array}\right.$
			•^{4} obtain simplified expression for z		•^{4} $(1+\lambda)z=3$, $z=\dfrac{3}{1+\lambda}$
			•^{5} statement based on expression at •^{4}		•^{5} $\lambda=-1$, accept $(\lambda\neq-1)$
Notes: 3.1 Row operations commentary not required for full credit. Methods not using augmented matrix may be acceptable.					

3.2 Not necessary to have unitary values for second elements for $\bullet^2$. 3.3 Accept lower triangular form. 3.4 If lower triangular form used, will need to have simplified expression for z. 3.5 Accept $z=\frac{-3}{-1-\lambda}$. 3.6 Also accept: when $\lambda=-1$ there are no solutions $\lambda<-1$ and $\lambda>-1$; $\lambda<-1$ or $\lambda>-1$.					
4	a		Ans: $3e^{3t}+2e^{t}$ $\bullet^1$ evidence of knowing to differentiate $\bullet^2$ complete differentiation	2	$\bullet^{1,2}$ $\frac{dy}{dx}=3e^{3t}\ldots$ $3e^{3t}+2e^{t}$
4	b		Ans: $\frac{38}{3}$ or $12\frac{2}{3}$ or equivalent $\bullet^3$ correctly set up integral $\bullet^4$ integrate and evaluate correctly	2	$\bullet^3$ $s=\int_0^{\ln 3} v\ dt=\int_0^{\ln 3}\left(e^{3t}+2e^{t}\right)dt$ $\bullet^4$ $=\left(\frac{1}{3}e^{3\ln 3}+2e^{\ln 3}\right)-\left(\frac{1}{3}+2\right)$ $\frac{38}{3}$ or $12\frac{2}{3}$ or equivalent
Note: 4.1 Accept rounded answers to 3 sf or better.					
5			Ans: $4\left(\cos\frac{2\pi}{3}+i\ \sin\frac{2\pi}{3}\right)$ $\bullet^1$ correct statement of conjugate $\bullet^2$ one of r,θ correct $\bullet^3$ complete substitution $\bullet^4$ process answer OR $\bullet^2$ obtain $\bar{z}^2$ in Cartesian form $\bullet^3$ one of r, θ correct	4	$\bullet^1$ $z=1+\sqrt{3}i$ $\bullet^{2,3}$ $\bar{z}=2\left(\cos\frac{\pi}{3}+i\ \sin\frac{\pi}{3}\right)$ $\bullet^4$ $\bar{z}^2=\left[2\left(\cos\frac{\pi}{3}+i\ \sin\frac{\pi}{3}\right)\right]^2=4\left(\cos\frac{2\pi}{3}+i\ \sin\frac{2\pi}{3}\right)$ $\bullet^2$ $\bar{z}^2=\left(1+\sqrt{3}i\right)^2=1+2\sqrt{3}i-3=-2+2\sqrt{3}i$ $\bar{z}^2=-2+2\sqrt{3}i=r(\cos\theta+i\sin\theta)$

			$\bullet^4$ second substitution		$\bullet^4$ $r=4$, $\theta=\frac{2\pi}{3}$, $\bar{z}^2=4\left(\cos\frac{2\pi}{3}+i\ \sin\frac{2\pi}{3}\right)$

Notes:

5.1 Accept $2cis\frac{\pi}{3}$ or the exponential form of a complex number – $\bar{z}=2e^{i\frac{\pi}{3}}$ at $\bullet^{2,3}$.

5.2 Accept angles expressed in degrees, ie $60°, 120°$.

5.3 Where a candidate has applied De Moivre's theorem to $k(\cos\theta - i\sin\theta)$ do not penalise.

5.4 Correct polar form only. Answer in form $k(\cos\theta - i\sin\theta)$ loses $\bullet^4$ unless correct form appears also.

5.5 Accept answers from $-\pi$ to 2π as being in polar form. For answers outside this range do not award $\bullet^4$.

6			Ans: $2y=-x+5$ or $y=-\frac{1}{2}x+\frac{5}{2}$ or $2y+x-5=0$	4	
			$\bullet^1$ x terms and constant		$\bullet^1$ $4x^3+\ldots 9-\ldots=0$
			$\bullet^2$ y terms		$\bullet^2$ $4x^3+4y^3\frac{dy}{dx}+9-6\frac{dy}{dx}=0$
			$\bullet^3$ gradient		$\bullet^3$ at $A(1,2)$ $\frac{dy}{dx}=\frac{4+9}{6-32}=\frac{-13}{26}=\frac{-1}{2}$
			$\bullet^4$ equation of tangent		$\bullet^4$ $y-2=-\frac{1}{2}(x-1)$

Notes:

6.1 Rearrangement and explicit statement of $\frac{dy}{dx}$ not required for full marks.

6.2 Where candidates assert that $\frac{d}{dx}(14)=14$, $\bullet^1$ not given, but $\bullet^2$, $\bullet^3$ and $\bullet^4$ all possible, leading to $26y=x+51$ (or equivalent) for $\left[\frac{3}{4}\right]$.

7	a		Ans: $\begin{pmatrix}16-2p & 5p\\ -10 & 1-2p\end{pmatrix}$	1	
			$\bullet^1$ correct answer		$\bullet^1$ $\begin{pmatrix}16-2p & 4p+p\\ -8-2 & -2p+1\end{pmatrix}=\begin{pmatrix}16-2p & 5p\\ -10 & 1-2p\end{pmatrix}$
7	b		Ans: $p=-2$	2	
			$\bullet^2$ property stated or implied (see note 7.4)		$\bullet^2$ A^2 is singular when det $A^2=0$
			$\bullet^3$ correct value for p (see note 7.1) OR $4+2p=0$ $p=-2$		$\bullet^3$ $p=-2$ OR $\bullet^2$ A^2 is singular when A is singular, ie when det $A=0$ Explicitly states property (not essential but preferred) $\bullet^3$ correct value of p (see note 7.1)

7	c		Ans: $x=12$ and $p=\frac{1}{3}$ $\bullet^4$ A transpose (A^T) correct. Does not have to be explicitly stated. $\bullet^5$ values of p **and** x correct	2	$\bullet^4$ $\begin{pmatrix} 4 & -2 \\ p & 1 \end{pmatrix}$ $\bullet^5$ $\begin{pmatrix} x & -6 \\ 1 & 3 \end{pmatrix} = 3\begin{pmatrix} 4 & -2 \\ p & 1 \end{pmatrix}$ $\begin{pmatrix} x & -6 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 12 & -6 \\ 3p & 3 \end{pmatrix}$ $x=12$ and $p=\frac{1}{3}$

Notes:

7.1 For (a) and (c), statement of answers only: award full marks. For (b), $p=-2$ only, award $\bullet^3$ only (1 out of 2).

7.2 Misinterpretation of A^T as inverse leading to $p=0$ and $x=\frac{3}{4}$ OR to $p=-\frac{8}{3}$ and $x=-\frac{9}{4}$ OR to $p=1$ and $x=\frac{1}{2}$.
OR any other set of inconsistent equations: do not award $\bullet^4$ or $\bullet^5$, ie 0 out of 2.
7.3 Accept unsimplified answers.
7.4 Usually implied by next line.
7.5 For any equation based on answer to (a), correctly obtaining all possible solutions, including complex, $\bullet^2$ $\bullet^3$ both available. 'No solutions', 'not possible' etc $\bullet^3$ not available, even if true.

8	a		Ans: $1-\frac{9x^2}{2}+\frac{27x^4}{8}$ $\bullet^1$ correct statement of series for $\cos x$ $\bullet^2$ substitute and evaluate coefficients	2	$\bullet^1$ $\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}\ldots$ $\bullet^2$ $\cos 3x = 1-\frac{(3x)^2}{2!}+\frac{(3x)^4}{4!}\ldots$ $=1-\frac{9x^2}{2}+\frac{81x^4}{24}\ldots$ $=1-\frac{9x^2}{2}+\frac{27x^4}{8}\ldots$
			OR $\bullet^1$ correct differentiation and evaluation if doing from first principles		$f(x)=\cos 3x \quad f(0)=1$ $f'(x)=-3\sin 3x \quad f'(0)=0$ $\bullet^1$ $f''(x)=-9\cos 3x \quad f''(0)=-9$ $f'''(x)=27\sin 3x \quad f'''(0)=0$ $f''''(x)=81\cos 3x \quad f''''(0)=81$
8	b		Ans: $e^{2x}=1+2x+2x^2+\frac{4x^3}{3}$	1	

			$\bullet^3$ state series with correct substitution		$\bullet^3$ $e^{2x} = 1 + 2x + 2x^2 + \frac{4x^3}{3}...$
8	c		Ans: $= 1 + 2x - \frac{5x^2}{2} - \frac{23x^3}{3}$ $\bullet^4$ know to multiply the two previously obtained series together $\bullet^5$ correctly multiply out brackets $\bullet^6$ simplify to lowest terms	3	$\bullet^4$ $e^{2x}\cos 3x = \left(1 - \frac{9x^2}{2} + \frac{27x^4}{8}...\right)\left(1 + 2x + 2x^2 + \frac{4x^3}{3}...\right)$ $\bullet^5$ $= 1 + 2x + 2x^2 + \frac{4x^3}{3} - \frac{9x^2}{2} - \frac{18x^3}{2}...$ $\bullet^6$ $= 1 + 2x - \frac{5x^2}{2} - \frac{23x^3}{3}...$
			OR $\bullet^4$ either all three derivatives correct OR first derivative and first two evaluations (above ....) OR all evaluations [not eased if at least one each of e^{2x} and $\sin/\cos 3x$] OR last two derivatives *and* last two evaluations correct $\bullet^5$ remainder correct $\bullet^6$ correct substitution of coefficients obtained at $\bullet^4$ into formula and simplify to lowest terms		$\bullet^4$ $f(x) = e^{2x}\cos 3x$ $\quad f(0) = 1$ $f'(x) = 2e^{2x}\cos 3x - 3e^{2x}\sin 3x$ $\quad f'(0) = 2$ .... $f''(x) = e^{2x}(-5\cos 3x - 12\sin 3x)$ $\quad f''(0) = -5$ $f'''(x) = e^{2x}(-46\cos 3x - 9\sin 3x)$ $\quad f'''(0) = -46$ $\bullet^5$ $e^{2x}\cos 3x = 1 + 2x + \frac{(-5)x^2}{2!} + \frac{(-46)x^3}{3!}...$ $e^{2x}\cos 3x = 1 + 2x - \frac{5x^2}{2} - \frac{46x^3}{6}...$ $\bullet^6$ $= 1 + 2x - \frac{5x^2}{2} - \frac{23x^3}{3}$
Notes: 8.1 Award $\bullet^1$ for substitution of $3x$ into series for $\cos x$. 8.2 Must have at least three terms for $\bullet^1$ if no further working. 8.3 Candidates may differentiate from first principles for any or all of the three required series for full credit. 8.4 For $\bullet^5$ and $\bullet^6$ ignore additional terms in x^4 or higher.					
9			Ans: proof $\bullet^1$ start proof	3	$\bullet^1$ (Given that x is irrational) 'assume that $\sqrt{x}$ is rational'

			$\bullet^2$ set up number and process		$\bullet^2$ $\sqrt{x}=\frac{a}{b}$ (a,b natural numbers with no common factor) $x=\frac{a^2}{b^2}$
			$\bullet^3$ complete proof		$\bullet^3$ x is rational, which is a contradiction, therefore the original statement is true
Notes:					
10			Ans: (0, 0) with justification	5	
			$\bullet^1$ find 1st derivative and begin to find 2nd derivative		$\bullet^1$ $\frac{dy}{dx}=\cos x+\sec^2 x$ $\frac{d^2y}{dx^2}=-\sin x+\ldots$
			$\bullet^2$ complete 2nd derivative		$\bullet^2$ $\frac{d^2y}{dx^2}=-\sin x+2(\cos x)^{-3}\sin x$ or equivalent
			$\bullet^3$ set 2nd derivative to zero and process		$\bullet^3$ $2\sin x-\sin x\cos^3 x=0$
			$\bullet^4$ solve to find potential POI		$\bullet^4$ $\sin x=0$, leads to $x=0, y=0$
			$\bullet^5$ check for 2nd derivative change of sign		$\bullet^5$ $\begin{array}{cccc} x & 0^- & 0 & 0^+ \\ \frac{d^2y}{dx^2} & - & 0 & + \end{array}$
Note: 10.1 $\bullet^5$ Candidates may refer to the curve's change of concavity.					
11			Ans: reflection in the line with equation $y=x$		
	a		$\bullet^1$correct reflection matrix M_1	1	$\bullet^1$ $\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$
	b		$\bullet^2$ correct rotation matrix M_2	1	$\bullet^2$ $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$
	c		$\bullet^3$ evaluation of M_3	1	$\bullet^3$ $\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$
	d		$\bullet^4$ correct transformation consistent with candidate's M_3	1	$\bullet^4$ reflection in the line with equation $y=x$
Notes:					

<table>
<tr><td>12</td><td></td><td></td><td>Ans: proof

●¹ test for $n=1$

●² state assumption

●³ consider $n=k+1$

●⁴ process

●⁵ complete proof</td><td>5</td><td>●¹ LHS $=\frac{1}{1\times 2}=\frac{1}{2}$ RHS $=1-\frac{1}{2}=\frac{1}{2}$
so true when $n=1$

●² assume true for $n=k,\ \sum_{r=1}^{k}\frac{1}{r(r+1)}=1-\frac{1}{k+1}$

●³ $\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\sum_{r=1}^{k}\frac{1}{r(r+1)}+1-\frac{1}{(k+1)(k+2)}$
$=1-\frac{1}{k+1}+\frac{1}{(k+1)(k+2)}$

●⁴ $=1-\frac{k+2-1}{(k+1)(k+2)}=1-\frac{k+1}{(k+1)\left((k+1)+1\right)}$

●⁵ Thus, if true for $n=k$, statement true for $n=k+1$, but since true for $n=1$, by mathematical induction, true for all $n\geq 1$.</td></tr>
<tr><td colspan="6">Notes:
12.1 Statement of conclusion can only gain ●⁵ if clear attempt at processing is shown at ●⁴.
12.2 Candidates may approach the question by stating the ‘target result’ with $n=k+1$ before starting processing for ●⁴. This is a valid method.</td></tr>
<tr><td>13</td><td></td><td></td><td>Ans: 9π units³

●¹ correctly identify circle equation

●² correct form of integral and limits

●³ substitute

●⁴ integrate

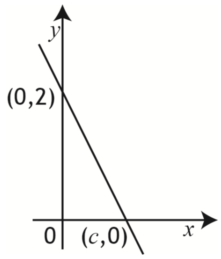
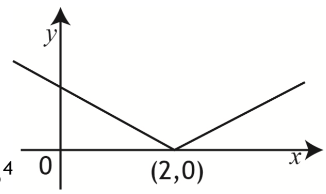
●⁵ evaluate</td><td>5</td><td>●¹ $(x-1)^2+y^2=4$ or equivalent

●² $V=\pi\int_0^3 y^2\ dx.$

●³ $=\pi\int_0^3\left(4-(x-1)^2\right)dx$

●⁴ $=\pi\left[4x-\frac{1}{3}(x-1)^3\right]_0^3$

●⁵ $=9\pi$ units³</td></tr>
<tr><td colspan="6">Notes:
13.1 Accept any version of circle equation.
13.2 Circle may be translated 1 unit left with appropriate equation and limits used.
13.3 For a numerical answer of $28\cdot 3$ or better to gain ●⁴ the result in terms of x must be shown.
13.4 ‘units³’ not required.</td></tr>
</table>

14	a		Ans: diagram ●[1] straight line with negative gradient crossing the positive sections of the x and y axes ●[2] both intersections correctly annotated	2	 ●[1,2]
14	b		Ans: $k=-c$ ●[3] correct value of k	1	●[3] $y=f(x)-c$ is odd therefore, $k=-c$
14	c		Ans: $h=2$ ●[4] sketch of $y=\lvert f(x)\rvert$ with point of reflection marked ●[5] explicit statement of answer	2	 ●[4] ●[5] $y=\lvert f(x+2)\rvert$ is even therefore, $h=2$
Notes:					
15			Ans: $\dfrac{1}{\sqrt{2}}$ ●[1] process solution to obtain both limits for θ ●[2] correctly replace all terms ●[3] replace $1+\tan^2\theta$ with $\sec^2\theta$ ●[4] simplify to integrable form ●[5] integrate **and** evaluate correctly	5	●[1] $x=1$ and $x=0$ become $\theta=\dfrac{\pi}{4}$ and $\theta=0$ ●[2] $\displaystyle\int_0^{\frac{\pi}{4}}\frac{\sec^2\theta\,d\theta}{\left(1+\tan^2\theta\right)^{\frac{3}{2}}}$ ●[3] $\displaystyle\int_0^{\frac{\pi}{4}}\frac{\sec^2\theta\,d\theta}{\left(\sec^2\theta\right)^{\frac{3}{2}}}$ ●[4] $\displaystyle\int_0^{\frac{\pi}{4}}\cos\theta\,d\theta$ ●[5] $\left[\sin\theta\right]_0^{\frac{\pi}{4}}=\dfrac{1}{\sqrt{2}}$
Note: 15.1 x limits can be kept during working provided integral expression is expressed back in terms of x.					

16	a		Ans: $\mathbf{r}_1 = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ $\mathbf{r}_2 = \begin{pmatrix} -5 \\ 2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 4 \\ 1 \end{pmatrix}$	2	
			•[1] L_1 equation correct (can be written using column vectors or $\mathbf{i}, \mathbf{j}, \mathbf{k}$)		•[1] $\boldsymbol{r}_1 = \boldsymbol{p} + \lambda \boldsymbol{u}_1$ where $\boldsymbol{p} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}$
			•[2] L_2 equation correct (can be written using column vectors or $\mathbf{i}, \mathbf{j}, \mathbf{k}$)		•[2] $\boldsymbol{r}_2 = \boldsymbol{q} + \mu \boldsymbol{u}_2$ where $\boldsymbol{q} = \begin{pmatrix} -5 \\ 2 \\ 5 \end{pmatrix}$
16	b		Ans: lines intersect at $(-1,\ -2,\ 4)$	4	
			•[3] two equations for two parameters		•[3] $2 + \lambda = -5 - 4\mu$ $\quad 4\mu + \lambda = -7$ $4 + 2\lambda = 2 + 4\mu$ $\quad \underline{4\mu - 2\lambda = 2}$
			•[4] two parameter solutions		•[4] $1 - \lambda = 5 + \mu$ $\quad \lambda = -3$ $\mu = -1$
			•[5] check third component in both equations		•[5] $z_1 = 1 - (-3)$ $\quad z_2 = 5 + (-1)$ $= 4$ $\quad = 4$
			•[6] point of intersection		•[6] Since $z_1 = z_2$, the lines intersect at $(-1, -2, 4)$
16	c		Ans: $2x + y + 4z = 12$ or equivalent	3	
			•[7] correct strategy to find normal		•[7] $\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & 2 & -1 \\ -4 & 4 & 1 \end{vmatrix}$ or $\begin{bmatrix} i & j & k & i & j \\ 1 & 2 & -1 & 1 & 2 \\ -4 & 4 & 1 & -4 & 4 \end{bmatrix}$
			•[8] normal vector		•[8] $= \boldsymbol{i}(2+4) - \boldsymbol{j}(1-4) + \boldsymbol{k}(4+8)$ $= 6\boldsymbol{i} + 3\boldsymbol{j} + 12\boldsymbol{k}$

			$\bullet^9$ find value for constant and equation		$\bullet^9$ $6x+3y+12z=\begin{pmatrix}6\\3\\12\end{pmatrix}\cdot\begin{pmatrix}\text{Point of}\\\text{intersection}\end{pmatrix}$ $=36$

Notes:

16.1 In (a), lines written in parametric or symmetric forms would gain only 1 mark out of 2 available.
16.2 In (c), the plane equation can be given in vector form, eg $\boldsymbol{r}=\boldsymbol{a}+s\boldsymbol{u}_1+t\boldsymbol{u}_2$ where $\boldsymbol{a}$ is position vector of a point on the plane and $s,t\in\boldsymbol{R}$.

17	a		Ans: $y=Ae^{-x}+Be^{2x}-\frac{1}{2}e^x-6$	7	
			$\bullet^1$ set up auxilary equation		$\bullet^1$ m^2-m-2
			$\bullet^2$ solutions to AE		$\bullet^2$ $(m-2)(m+1)=0$ $m=-1$ or $m=2$
			$\bullet^3$ state complementary function		$\bullet^3$ $y=Ae^{-x}+Be^{2x}$
			$\bullet^4$ state particular integral		$\bullet^4$ $y=Ce^x+D$
			$\bullet^5$ process		$\bullet^5$ $\frac{dy}{dx}=Ce^x\Rightarrow\frac{d^2y}{dx^2}=Ce^x$
			$\bullet^6$ calculate C and D		$\bullet^6$ $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=e^x+12$ $\left[Ce^x\right]-\left[Ce^x\right]-2\left[Ce^x+D\right]=e^x+12$ $-2Ce^x-2D=e^x+12$ Hence, $C=-\frac{1}{2}$ and $D=-6$
			$\bullet^7$ state general solution		$\bullet^7$ $y=Ae^{-x}+Be^{2x}-\frac{1}{2}e^x-6$
17	b		Ans: $y=3e^{-x}+2e^{2x}-\frac{1}{2}e^x-6$	3	
			$\bullet^8$ set up equations		$\bullet^8$ $x=0$ and $y=-\frac{3}{2}\Rightarrow$ $A+B-\frac{1}{2}-6=-\frac{3}{2}$ $x=0$ and $\frac{dy}{dx}=\frac{1}{2}\Rightarrow$ $-A+2B-\frac{1}{2}=\frac{1}{2}$

			•⁹ process to find A and B •¹⁰ state particular solution		•⁹ $3B-7=-1 \Rightarrow B=2 \Rightarrow A=3$ •¹⁰ $y=3e^{-x}+2e^{2x}-\frac{1}{2}e^{x}-6$
Notes:					
18	a		Ans: proof •¹ express related rates of change •² complete 'show that'	2	Method 1 $V = Ah$ $\frac{dh}{dt}=\frac{dh}{dV}.\frac{dV}{dt}$ •¹ $\frac{dV}{dh}=$ A* $\therefore \frac{dh}{dV}=\frac{1}{\text{A}}$* $=\frac{1}{A}.-k\sqrt{h}$ •² $=\frac{-k}{A}\sqrt{h}$ or Method 2 $V = Ah$ $\frac{dV}{dt}=\frac{d}{dt}(Ah)$ •¹ $-k\sqrt{h}=A\frac{dh}{dt}$ $\frac{Adh}{dt}=-k\sqrt{h}$ •² $\frac{dh}{dt}=\frac{-k}{A}\sqrt{h}$
18	b		Ans: proof •³ substitute values •⁴ separate variables •⁵ integrate correctly	4	$\frac{dh}{dt}=-0\cdot3$ cm/hr when $h=144$ •³ $-0\cdot3=-\frac{k}{A}\sqrt{144}$ $\frac{k}{A}=\frac{1}{40}\therefore A=40k$ •⁴ $\int\frac{1}{\sqrt{h}}dh=\int\frac{-k}{A}dt$ or $\int\frac{1}{\sqrt{h}}dh=\int-\frac{1}{40}dt$ •⁵ $2\sqrt{h}=\frac{-k}{A}t+c$

			•[6] evaluate constant and complete rearrangement		•[6] $2\sqrt{144}=c$ $c=24$ $2\sqrt{h}=\frac{-k}{A}t+24$ $\sqrt{h}=\frac{-k}{2A}t+12$ $h=\left(\frac{-k}{2A}t+12\right)^2$ $h=\left(\frac{-1}{80}t+12\right)^2$
18	c		Ans: 40 days •[7] know to set to zero •[8] calculate the number of days	2	•[7] $0=\left(-\frac{1}{80}t+12\right)^2$ $t=960$ hours •[8] number days $=\frac{960}{24}=40$ days
18	d		Ans: rate to vegetation is $108\pi(\text{cm}^3/\text{hr})$ •[9] find k •[10] calculate h or $\sqrt{h}$ •[11] process to and interpret answer for $\frac{dV}{dt}$	3	•[9] $A=400\pi$, so $\frac{k}{A}=\frac{1}{40}$ $k=10\pi$ •[10] $h=\left(\frac{-1}{80}.96+12\right)^2=\left(\frac{54}{5}\right)^2$ •[11] $\frac{dV}{dt}=-108\pi[\text{cm}^3/\text{hr}]$

Notes:

18.1 In (a), one or both of the * lines needed for method 1.
18.2 In (c), accept any numerical answer rounding to 339. Do not penalise the omission of units.
18.3 In (c), accept the omission of a negative sign for $\frac{dV}{dt}$ provided interpretation, eg as in the answer above demonstrates understanding of the context of the problem.
18.4 $A=\frac{dV}{dt}=119\cdot5\pi\,\text{cm}^3/\text{hr}$ which comes from taking $t=4$. Do not award •[10].
18.5 Using $h=144$ in part (d) leading to 377, do not award •[10] or •[11].
18.6 Do not penalise the omission of integration symbols.
18.7 Where candidates use 144 instead of 0 initially, •[7] lost, but •[8] available if resulting quadratic solved correctly to obtain both $t=0$ **and** $t=1920$, discarding $t=0$ answer *and* converting to 80 days.

[END OF EXEMPLAR MARKING INSTRUCTIONS]