



National
Qualifications
EXEMPLAR PAPER ONLY

EP30/H/01

**Mathematics
Paper 1
(Non-Calculator)**

Marking Instructions

These Marking Instructions have been provided to show how SQA would mark this Exemplar Question Paper.

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General Marking Principles for Higher Mathematics

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- (a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.
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Definitions of Mathematics-specific command words used in this Paper are:

Determine: obtain an answer from given facts, figures or information;

Expand: multiply out an algebraic expression by making use of the distributive law or a compound trigonometric expression by making use of one of the addition formulae for $\sin(A \pm B)$ or $\cos(A \pm B)$;

Express: use given information to rewrite an expression in a specified form;

Find: obtain an answer showing relevant stages of working;

Hence: use the previous answer to proceed;

Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used;

Identify: provide an answer from a number of possibilities;

Justify: show good reason(s) for the conclusion(s) reached;

Show that: use mathematics to prove something, eg that a statement or given value is correct – all steps, including the required conclusion, must be shown;

Sketch: give a general idea of the required shape or relationship and annotate with all relevant points and features;

Solve: obtain the answer(s) using algebraic and/or numerical and/or graphical methods.

Detailed Marking Instructions for each question

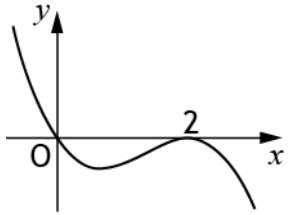
Question	Expected response (Give one mark for each •)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each •)
1	$y - 12 = 6(x - 5)$	3	
	<ul style="list-style-type: none"> •¹ know to differentiate •² calculate gradient •³ state equation of tangent 		<ul style="list-style-type: none"> •¹ $2x - 4$ •² 6 •³ $y - 12 = 6(x - 5)$
2	$a = 1, b = -2$ and $k = -1$	3	
	<ul style="list-style-type: none"> •¹ interpret a and b •² know to substitute (1, 2) •³ state the value of k 		<ul style="list-style-type: none"> •¹ $a = 1, b = -2$ or $a = -2, b = 1$ •² $2 = k \times 1 \times (1 + 1) \times (1 - 2)$ •³ -1
3	$\frac{1}{12}$	3	
	<ul style="list-style-type: none"> •¹ complete integration •² substitute limits •³ evaluate 		<ul style="list-style-type: none"> •¹ $-\frac{1}{6}x^{-1}$ •² $\left(-\frac{1}{6 \times 2}\right) - \left(-\frac{1}{6 \times 1}\right)$ •³ $\frac{1}{12}$
4	Statements B and D are true.	3	
	<ul style="list-style-type: none"> •¹ statements B and D correct •² calculate maximum value •³ calculate value of x 		<ul style="list-style-type: none"> •¹ B and D •² max is $2 - 3 \times -1$ or $f\left(\frac{11\pi}{6}\right) = 2 - 3\sin\left(\frac{11\pi}{6} - \frac{\pi}{3}\right) = 2 - 3\sin\left(\frac{3\pi}{2}\right) = 5$ •³ $x - \frac{\pi}{3} = \frac{3\pi}{2} \Rightarrow x = \frac{3\pi}{2} + \frac{\pi}{3} \Rightarrow x = \frac{11\pi}{6}$

5	(a)	$a = -7$ and $b = 10$	4	
		<ul style="list-style-type: none"> •¹ know to use $x = 1$ and obtain an equation •² know to use $x = 2$ and obtain an equation •³ process equations to find one value •⁴ find the other value 		<ul style="list-style-type: none"> •¹ $(1)^3 - 4(1)^2 + a(1) + b = 0$ •² $(2)^3 - 4(2)^2 + a(2) + b = -12$ •³ $a = -7$ and $b = 10$ •⁴ $b = 10$ and $a = -7$
Notes		1	An incorrect value at • ³ should be followed through for the possible award of • ⁴ . However, if the equations are such that no solution exists, then • ³ and • ⁴ are not available.	
		2	Synthetic Division is an acceptable alternative method.	
5	(b)	$x = 1, x = 5, x = -2$	4	
		<ul style="list-style-type: none"> •⁵ substitute for a and b and know to divide by $x - 1$ •⁶ obtain quadratic factor •⁷ complete factorisation •⁸ state solution 		<ul style="list-style-type: none"> •⁵ $(x^3 - 4x^2 - 7x + 10) \div (x - 1)$ stated or implied by •⁶ •⁶ $(x - 1)(x^2 - 3x - 10)$ •⁷ $(x - 1)(x - 5)(x + 2)$ •⁸ $x = 1, x = 5, x = -2$
Notes		3	For candidates who substitute $a = -7$ into the correct quotient from part (a), • ⁵ , • ⁶ and • ⁷ are available.	
		4	Candidates who use incorrect values obtained in part (a) may gain • ⁵ , • ⁶ and • ⁷ .	
		5	Where the quadratic factor obtained is irreducible, candidates must clearly demonstrate that $b^2 - 4ac < 0$ to gain mark • ⁷ .	
		6	Do not penalise the inclusion of “= 0” or for solving for x .	
		7	Candidates who use values, ex nihilo, for a and b can gain • ⁵ , if division is correct.	

6	(a)	$y - 3 = \frac{1}{3}(x - 1)$	4	
		<ul style="list-style-type: none"> •¹ find midpoint of PQ •² find gradient of PQ •³ interpret perpendicular gradient •⁴ state equation of perpendicular bisector 		<ul style="list-style-type: none"> •¹ (1, 3) •² -3 •³ $\frac{1}{3}$ •⁴ $y - 3 = \frac{1}{3}(x - 1)$
Notes		<p>1 •⁴ is only available if a midpoint and a perpendicular gradient are used.</p> <p>2 Candidates who use $y = mx + c$ must obtain a numerical value for c before •⁴ is available.</p>		
6	(b)	$y - (-2) = -3(x - 1)$	2	
		<ul style="list-style-type: none"> •⁵ use parallel gradients •⁶ state equation of line 		<ul style="list-style-type: none"> •⁵ -3 •⁶ $y - (-2) = -3(x - 1)$
Notes		<p>3 •⁶ is only available to candidates who use R and their gradient of PQ from (a).</p>		
6	(c)	$x = -\frac{1}{2}, y = \frac{5}{2}$	3	
		<ul style="list-style-type: none"> •⁷ use valid approach •⁸ solve for one variable •⁹ solve for other variable 		<ul style="list-style-type: none"> •⁷ $x - 3y = -8$ and $9x + 3y = 3$ or $-3x + 1 = \frac{1}{3}x + \frac{8}{3}$ or $3(3y - 8) + y = 1$ •⁸ $x = -\frac{1}{2}$ •⁹ $y = \frac{5}{2}$
Notes		<p>4 Neither $x - 3y = -8$ and $3x + y = 1$ nor $y = -3x + 1$ and $3y = x + 8$ are sufficient to gain •⁷.</p> <p>5 •⁷, •⁸ and •⁹ are not available to candidates who:</p> <ul style="list-style-type: none"> — equate zeros — give answers only, without working — use R for equations in both (a) and (b) — use the same gradient for the lines in (a) and (b) 		

6	(d)	$\frac{\sqrt{5}}{\sqrt{2}}$ <ul style="list-style-type: none"> •¹⁰ identify appropriate points •¹¹ calculate distance 	2	<ul style="list-style-type: none"> •¹⁰ (1, 3) and $\left(-\frac{1}{2}, \frac{5}{2}\right)$ •¹¹ $\frac{\sqrt{5}}{\sqrt{2}}$ accept $\frac{\sqrt{10}}{2}$ or $\sqrt{2 \cdot 5}$
Notes		<p>6 •¹⁰ and •¹¹ are only available for considering the distance between the midpoint of PQ and the candidate's answer from (c) or for considering the perpendicular distance from P or Q to l_2.</p> <p>7 At least one coordinate at •¹⁰ stage must be a fraction for •¹¹ to be available.</p> <p>8 There should only be one calculation of a distance to gain •¹¹.</p>		
7	(a)	0, 60, 300 <ul style="list-style-type: none"> •¹ know to use double angle formula •² express as a quadratic in $\cos x^\circ$ •³ start to solve •⁴ reduce to equations in $\cos x^\circ$ only •⁵ process solutions in given domain 	5	<p>Method 1: Using factorisation</p> <ul style="list-style-type: none"> •¹ $2 \cos^2 x^\circ - 1 \dots$ stated or implied by •² •² $2 \cos^2 x^\circ - 3 \cos x^\circ + 1 = 0$ } = 0 must appear at either of these lines to gain •² •³ $(2 \cos x^\circ - 1)(\cos x^\circ - 1)$ } <p>Method 2: Using quadratic formula</p> <ul style="list-style-type: none"> •¹ $2 \cos^2 x^\circ - 1 \dots$ stated or implied by •² •² $2 \cos^2 x^\circ - 3 \cos x^\circ + 1 = 0$ stated explicitly •³ $\frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times 1}}{2 \times 2}$ <p>In both methods:</p> <ul style="list-style-type: none"> •⁴ $\cos x^\circ = \frac{1}{2}$ and $\cos x^\circ = 1$ •⁵ 0, 60, 300 Candidates who include 360 lose •⁵. <p>or</p> <ul style="list-style-type: none"> •⁴ $\cos x = 1$ and $x = 0$ •⁵ $\cos x^\circ = \frac{1}{2}$ and $x = 60$ or 300 Candidates who include 360 lose •⁵.
Notes		<p>1 •¹ is not available for simply stating that $\cos 2A = 2 \cos^2 A - 1$ with no further working.</p> <p>2 In the event of $\cos^2 x - \sin^2 x$ or $1 - 2 \sin^2 x$ being substituted for $\cos 2x$, •¹ cannot</p>		

		<p>be awarded until the equation reduces to a quadratic in $\cos x$.</p> <p>3 Substituting $\cos 2A = 2\cos^2 A - 1$ or $\cos 2a = 2\cos^2 a - 1$ etc should be treated as bad form throughout.</p> <p>4 Candidates may express the quadratic equation obtained at the \bullet^2 stage in the form $2c^2 - 3c + 1$ or $2x^2 - 3x + 1$ etc. For candidates who do not solve a trigonometric quadratic equation at \bullet^5, $\cos x$ must appear explicitly to gain \bullet^4.</p> <p>5 \bullet^4 and \bullet^5 are only available as a consequence of solving a quadratic equation.</p> <p>6 Any attempt to solve $ax^2 + bx = c$ loses \bullet^3, \bullet^4 and \bullet^5.</p> <p>7 \bullet^5 is not available to candidates who work in radian measure and do not convert their answers into degree measure.</p>	
7	(b)	0, 30, 150, 180, 210 and 330	2
		<ul style="list-style-type: none"> \bullet^6 interpret relationship with (a) \bullet^7 state valid values 	<ul style="list-style-type: none"> \bullet^6 $2x = 0$ and 60 and 300 \bullet^7 0, 30, 150, 180, 210 and 330
Notes		<p>8 Do not penalise the inclusion of 360 in (b).</p> <p>9 Ignore extra answers, correct or incorrect, outside the given interval, but penalise incorrect answers within the interval.</p> <p>10 Do not penalise candidates who use radians in (b) if they have already been penalised in (a).</p> <p>11 Candidates who go back to “first principles” for (b) can only gain \bullet^6 and \bullet^7 for a correct method leading to valid solutions.</p>	
8	(a)		3
		<ul style="list-style-type: none"> \bullet^1 reflection in x-axis \bullet^2 translation $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ \bullet^3 annotation of “transformed” graph 	<ul style="list-style-type: none"> \bullet^1 reflection of graph in x-axis \bullet^2 graph moves parallel to y-axis by 2 units upwards \bullet^3 two “transformed” points appropriately annotated

Notes	<p>1 All graphs must include both the x and y axes (labelled or unlabelled), however the origin need not be labelled.</p> <p>2 No marks are available unless a graph is attempted.</p> <p>3 No marks are available to a candidate who makes several attempts at a graph on the same diagram, unless it is clear which is the final graph.</p> <p>4 A linear graph gains no marks in both (a) and (b).</p> <p>5 For ●³ “transformed” means a reflection followed by a translation.</p> <p>6 ●¹ and ●² apply to the entire curve.</p> <p>7 A reflection in any line parallel to the y-axis does not gain ●¹ or ●³.</p> <p>8 A translation other than $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ does not gain ●² or ●³.</p>	
8	<p>(b)</p>  <p>●⁴ identify roots</p> <p>●⁵ interpret point of inflection</p> <p>●⁶ complete cubic curve</p>	<p>3</p> <p>●⁴ 0 and 2 only</p> <p>●⁵ turning point at (2, 0)</p> <p>●⁶ cubic passing through origin with negative gradient</p>
9	<p>(a)</p> <p>$k = 2$ and $a = \frac{\pi}{3}$</p> <p>●¹ use appropriate compound angle formula</p> <p>●² compare coefficients</p> <p>●³ process k</p> <p>●⁴ process a</p>	<p>4</p> <p>●¹ $k \cos A \cos B - k \sin A \sin B$ stated explicitly</p> <p>●² $k \cos a = 1$ and $k \sin a = \sqrt{3}$ stated explicitly</p> <p>●³ 2 (do not accept $\sqrt{4}$)</p> <p>●⁴ $\frac{\pi}{3}$ but must be consistent with ●²</p>
Notes	<p>1 Treat $k \cos A \cos B - \sin A \sin B$ as bad form only if the equations at the ●² stage both contain k.</p> <p>2 $2 \cos A \cos B - 2 \sin A \sin B$ or $2(\cos A \cos B - \sin A \sin B)$ is acceptable for ●¹ and ●³.</p> <p>3 Accept $k \cos a = 1$ and $-k \sin a = -\sqrt{3}$ for ●².</p> <p>4 ●² is not available for $k \cos 4x = 1$ and $k \sin 4x = \sqrt{3}$, however, ●⁴ is still available.</p> <p>5 ●⁴ is only available for a single value of a.</p> <p>6 Candidates who work in degrees and do not convert to radian measure in (a) do not gain ●⁴.</p>	

	7	Candidates may use any form of the wave equation for ● ¹ , ● ² and ● ³ , however, ● ⁴ is only available if the value of a is interpreted for the form $k \cos(4x+a)$.
9	(b)	$\left(\frac{\pi}{24}, 0\right) \left(\frac{7\pi}{24}, 0\right)$ ● ⁵ strategy for finding roots ● ⁶ start to solve for multiple angles ● ⁷ state both roots in given domain
	3	● ⁵ $2 \cos\left(4x + \frac{\pi}{3}\right) = 0$ or $\sqrt{3} \sin 4x = \cos 4x$ ● ⁶ $4x = \left(\frac{\pi}{2} - \frac{\pi}{3}\right), \left(\frac{3\pi}{2} - \frac{\pi}{3}\right) \dots$ ● ⁷ $\frac{\pi}{24}, \frac{7\pi}{24}$
Notes	8	Candidates should only be penalised once for leaving their answer in degrees in (a) and (b).
	9	If the expression used in (b) is not consistent with (a) then only ● ⁶ and ● ⁷ are available.
	10	Correct roots without working cannot gain ● ⁶ but will gain ● ⁷ .
	11	Candidates should only be penalised once for not simplifying $\sqrt{4}$ in (a) and (b).
10		$y = \frac{3}{2} \sin 2x + \frac{\sqrt{3}}{4}$ ● ¹ know to integrate ● ² substitute $\left(\frac{7\pi}{6}, \sqrt{3}\right)$ ● ³ use exact values ● ⁴ express y in terms of x
	4	● ¹ $\frac{3}{2} \sin 2x + \dots$ ● ² $\sqrt{3} = \frac{3}{2} \sin\left(2 \times \frac{7\pi}{6}\right) + c$ ● ³ $\sqrt{3} = \frac{3}{2} \times \left(\frac{\sqrt{3}}{2}\right) + c$ ● ⁴ $y = \frac{3}{2} \sin 2x + \frac{\sqrt{3}}{4}$
11	(a)	$3(x^3 - 1) + 1$ ● ¹ interpret notation ● ² complete process
	2	● ¹ $g(x^3 - 1)$ ● ² $3(x^3 - 1) + 1$

11	(b)	$h(x) = \sqrt[3]{\frac{x+2}{3}}$	3	
		<ul style="list-style-type: none"> •³ start to rearrange for $x =$ •⁴ rearrange •⁵ write in functional form: $h(x) =$ or $y =$ 		<ul style="list-style-type: none"> •³ $3x^3 = y + 2$ •⁴ $x = \sqrt[3]{\frac{y+2}{3}}$ •⁵ $h(x) = \sqrt[3]{\frac{x+2}{3}}$

[END OF EXEMPLAR MARKING INSTRUCTIONS]



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**Mathematics
Paper 2**

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Express: use given information to rewrite an expression in a specified form;

Find: obtain an answer showing relevant stages of working;

Hence: use the previous answer to proceed;

Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used;

Identify: provide an answer from a number of possibilities;

Justify: show good reason(s) for the conclusion(s) reached;

Show that: use mathematics to prove something, eg that a statement or given value is correct – all steps, including the required conclusion, must be shown;

Sketch: give a general idea of the required shape or relationship and annotate with all relevant points and features;

Solve: obtain the answer(s) using algebraic and/or numerical and/or graphical methods.

Detailed Marking Instructions for each question

Question		Expected Response (Give one mark for each •)	Max mark	Additional Guidance (Illustration of evidence for awarding a mark at each •)
1	(a)	$u_1 = 8$ and $u_2 = -4$	1	
		• ¹ find terms of sequence		• ¹ $u_1 = 8$ and $u_2 = -4$
1	(b)	$p = 2$ or $q = -3$	3	
		• ² interpret sequence		• ² eg $4p + q = 5$ and $5p + q = 7$
		• ³ solve for one variable		• ³ $p = 2$ or $q = -3$
		• ⁴ state second variable		• ⁴ $q = -3$ or $p = 2$
Notes		1 Candidates may use $7p + q = 11$ as one of their equations at • ² .		
		2 Treat equations like $p4 + q = 5$ or $p(4) + q = 5$ as bad form.		
		3 Candidates should not be penalised for using $u_{n+1} = pu_n + q$.		
1	(c)	(i)	3	
				• ⁵ know how to find a valid limit
		• ⁶ calculate a valid limit only		• ⁶ $l = 0$
	(ii)	• ⁷ state reason		• ⁷ outside interval $-1 < p < 1$
Notes		4 Just stating that $l = al + b$ or $l = \frac{b}{1-a}$ is not sufficient for • ⁵ .		
		5 Any calculations based on formulae masquerading as a limit rule cannot gain • ⁵ and • ⁶ .		
		6 For candidates who use “ $b=0$ ”, • ⁶ is only available to those who simplify $\frac{0}{\dots}$ to 0.		
		7 Accept $2 > 1$ or $p > 1$ for • ⁷ . This may be expressed in words.		
		8 Candidates who use a without reference to p or 2 cannot gain • ⁷ .		

2	(a)	P (-3, -1) Q (1, 7)	6	<p>Substituting for y</p> <ul style="list-style-type: none"> •¹ $y = 2x + 5$ stated or implied by •² •² $\dots(2x + 5)^2 \dots - 2(2x + 5)\dots$ •³ $5x^2 + 10x - 15 = 0$ } = 0 must appear at the •³ •⁴ eg $5(x + 3)(x - 1)$ } or •⁴ stage to gain •³ •⁵ $x = -3$ and $x = 1$ •⁶ $y = -1$ and $y = 7$ <p>Substituting for x</p> <ul style="list-style-type: none"> •¹ $x = \frac{y - 5}{2}$ stated or implied by •² •² $\left(\frac{y - 5}{2}\right)^2 \dots - 6\left(\frac{y - 5}{2}\right)\dots$ •³ $5y^2 - 30y - 35 = 0$ } = 0 must appear at the •³ •⁴ eg $5(y + 1)(y - 7)$ } or •⁴ stage to gain •³ •⁵ $y = -1$ and $y = 7$ •⁶ $x = -3$ and $x = 1$
Notes		<p>1 At •⁴ the quadratic must lead to two real distinct roots for •⁵ and •⁶ to be available.</p> <p>2 Cross marking is available here for •⁵ and •⁶.</p> <p>3 Candidates do not need to distinguish between points P and Q.</p>		

2	<p>(b) $(x+5)^2 + (y-5)^2 = 40$</p> <ul style="list-style-type: none"> •⁷ centre of original circle •⁸ radius of original circle <p>Method 1: Using midpoint</p> <ul style="list-style-type: none"> •⁹ midpoint of chord •¹⁰ evidence for finding new centre •¹¹ centre of new circle •¹² equation of new circle <p>Method 2: Stepping out using P and Q</p> <ul style="list-style-type: none"> •⁹ evidence of C_1 to P or C_1 to Q •¹⁰ evidence of Q to C_2 or P to C_2 •¹¹ centre of new circle •¹² equation of new circle 	6	<ul style="list-style-type: none"> •⁷ (3, 1) •⁸ $\sqrt{40}$ accept $r^2 = 40$ <p>Method 1: Using midpoint</p> <ul style="list-style-type: none"> •⁹ (-1, 3) •¹⁰ eg stepping out or midpoint formula •¹¹ (-5, 5) •¹² $(x+5)^2 + (y-5)^2 = 40$ <p>Method 2: Stepping out using P and Q</p> <ul style="list-style-type: none"> •⁹ eg stepping out or vector approach •¹⁰ eg stepping out or vector approach •¹¹ (-5, 5) •¹² $(x+5)^2 + (y-5)^2 = 40$
Notes	<p>4 The evidence for •⁷ and •⁸ may appear in (a).</p> <p>5 Centre (-5, 5) without working in method 1 may still gain •¹² but not •¹⁰ or •¹¹, in method 2 may still gain •¹² but not •⁹, •¹⁰ or •¹¹. Any other centre without working in method 1 does not gain •¹⁰, •¹¹ or •¹², in method 2 does not gain •⁹, •¹⁰, •¹¹ or •¹².</p> <p>6 The centre must have been clearly indicated before it is used at the •¹² stage.</p> <p>7 Do not accept, eg $\sqrt{40}^2$ or 39.69, or any other approximations for •¹².</p> <p>8 The evidence for •⁸ may not appear until the candidate states the radius or equation of the second circle.</p>		
3	<p>$-7 < p < 5$</p> <ul style="list-style-type: none"> •¹ substitute into discriminant •² know condition for no real roots •³ factorise •⁴ solve for p 	4	<ul style="list-style-type: none"> •¹ $(p+1)^2 - 4 \times 1 \times 9$ •² $b^2 - 4ac < 0$ •³ $(p-5)(p+7) < 0$ •⁴ $-7 < p < 5$

4		$\frac{27}{4}$	5	<ul style="list-style-type: none"> •¹ $\int_{-3}^0 \dots \dots \dots$ •² $\int_{-3}^0 (x^3 + 3x^2 + 2x + 3) - (2x + 3) dx$ •³ $\frac{1}{4}x^4 + x^3$ •⁴ $0 - \left(\frac{1}{4}(-3)^4 + (-3)^3 \right)$ •⁵ $\frac{27}{4} \text{ units}^2$
Notes		<p>1 Where a candidate differentiates one or more terms at •³ then •⁴ and •⁵ are not available.</p> <p>2 Candidates who substitute without integrating at •² do not gain •³, •⁴ and •⁵.</p> <p>3 Candidates must show evidence that they have considered the upper limit 0 at •⁴.</p> <p>4 Where candidates show no evidence for both •³ and •⁴, but arrive at the correct area, then •³, •⁴ and •⁵ are not available.</p> <p>5 The omission of dx at •² should not be penalised.</p>		
5	(a)	$\overline{OB} = 4\mathbf{i} + 4\mathbf{j}$	1	<ul style="list-style-type: none"> •¹ $4\mathbf{i} + 4\mathbf{j}$
5		<p>(b)</p> $\overline{DB} = \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$ $\overline{DM} = \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$	3	<ul style="list-style-type: none"> •² $\begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$ •³ $(2, 0, 0)$ stated, or implied by •⁴ •⁴ $\begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$
		<ul style="list-style-type: none"> •² state components of \overline{DB} •³ state coordinates of M •⁴ state components of \overline{DM} 		

5	(c)	<p>40 · 3° or 0 · 703 rads</p> <ul style="list-style-type: none"> •⁵ know to use scalar product •⁶ find scalar product •⁷ find magnitude of a vector •⁸ find magnitude of a vector •⁹ evaluate angle BDM 	5	<ul style="list-style-type: none"> •⁵ $\cos \hat{BDM} = \frac{\overrightarrow{DB} \cdot \overrightarrow{DM}}{ \overrightarrow{DB} \cdot \overrightarrow{DM} }$ stated or implied by •⁹ •⁶ $\overrightarrow{DB} \cdot \overrightarrow{DM} = 32$ •⁷ $\overrightarrow{DB} = \sqrt{44}$ •⁸ $\overrightarrow{DM} = \sqrt{40}$ •⁹ 40 · 3° or 0 · 703 rads
Notes		<p>1 •⁵ is not available to candidates who evaluate the wrong angle. 2 If candidates do not attempt •⁹, then •⁵ is only available if the formula quoted relates to the labelling in the question. 3 •⁹ should be awarded to any answer which rounds to 40° or 0 · 7 radians. 4 In the event that both magnitudes are equal or there is only one non-zero component, •⁸ is not available.</p>		
6		<p>$\frac{27}{2}$</p> <ul style="list-style-type: none"> •¹ use distributive law •² calculate scalar product •³ calculate scalar product •⁴ process scalar product = 0 and complete 	4	<ul style="list-style-type: none"> •¹ $\mathbf{p} \cdot \mathbf{p} + \mathbf{p} \cdot \mathbf{q} + \mathbf{p} \cdot \mathbf{r}$ •² $\mathbf{p} \cdot \mathbf{p} = 9$ •³ $\mathbf{p} \cdot \mathbf{q} = \frac{9}{2}$ •⁴ $\mathbf{p} \cdot \mathbf{r} = 0$ and $\frac{27}{2}$
7	(a)	<p>$k \approx 0 \cdot 028$</p> <ul style="list-style-type: none"> •¹ interpret half-life •² process equation •³ write in logarithmic form •⁴ process for k 	4	<ul style="list-style-type: none"> •¹ $\frac{1}{2} P_0 = P_0 e^{-25k}$ stated or implied by •² •² $e^{-25k} = \frac{1}{2}$ •³ $\log_e \frac{1}{2} = -25k$ •⁴ $k \approx 0 \cdot 028$
Notes		<p>1 Do not penalise candidates who substitute a numerical value for P_0 in part (a).</p>		

7	(b)	No, with reason	4	
		<ul style="list-style-type: none"> •⁵ interpret equation •⁶ process •⁷ state percentage decrease •⁸ justify answer 	<ul style="list-style-type: none"> •⁵ $P_t = P_0 e^{-80 \times 0.028}$ •⁶ $P_t \approx 0.1065 P_0$ •⁷ 89% •⁸ No, the concentration will not have decreased by over 90%. 89% decrease. 	
Notes		<p>2 For candidates who use a value of k which does not round to 0.028, •⁵ is not available unless already penalised in part (a).</p> <p>3 For a value of k ex-nihilo then •⁵, •⁶ and •⁷ are not available.</p> <p>4 •⁶ is only available for candidates who express P_t as a multiple of P_0.</p> <p>5 Beware of candidates using proportion. This is not a valid strategy.</p>		
8	$\frac{3\pi}{8}$ <ul style="list-style-type: none"> •¹ start to integrate •² complete integration •³ process limits •⁴ simplify numeric term and equate to $\frac{10}{4}$ •⁵ start to solve equation •⁶ solve for a 		6	<ul style="list-style-type: none"> •¹ $-\frac{5}{4} \cos \dots$ •² $-\frac{5}{4} \cos\left(4x - \frac{\pi}{2}\right)$ •³ $-\frac{5}{4} \cos\left(4a - \frac{\pi}{2}\right) + \frac{5}{4} \cos\left(\frac{4\pi}{8} - \frac{\pi}{2}\right)$ •⁴ $-\frac{5}{4} \cos\left(4a - \frac{\pi}{2}\right) + \frac{5}{4} = \frac{10}{4}$ •⁵ $\cos\left(4a - \frac{\pi}{2}\right) = -1$ •⁶ $a = \frac{3\pi}{8}$
Notes		<p>1 Candidates who include solutions outwith the range cannot gain •⁶.</p> <p>2 The inclusion of $+c$ at •¹ or •² should be treated as bad form.</p> <p>3 •⁶ is only available for a valid numerical answer.</p> <p>4 Where the candidate differentiates, •¹, •² and •³ are not available.</p> <p>5 Where the candidate integrates incorrectly, •³, •⁴, •⁵ and •⁶ are still available.</p> <p>6 The value of a must be given in radians.</p>		

9	(a)	4 cm	5	
		<ul style="list-style-type: none"> •¹ prepare to differentiate •² differentiate •³ equate derivative to 0 •⁴ process for x •⁵ verify nature 		<ul style="list-style-type: none"> •¹ ... $48x^{-1}$ •² $3 - 48x^{-2}$ •³ $3 - 48x^{-2} = 0$ •⁴ $x = 4$ •⁵ nature table or 2nd derivative
Notes		1 Do not penalise the non-appearance of -4 at • ⁴ .		
9	(b)	No, (£198 > £195)	2	
		<ul style="list-style-type: none"> •⁶ evaluate L •⁷ calculate cost and justify answer 		<ul style="list-style-type: none"> •⁶ $L = 24$ •⁷ $24 \times £8 \cdot 25 = £198$. No and reason (£198 > £195)
Notes		2 Candidates who process $x = -4$ to obtain $L = -24$ do not gain • ⁶ . 3 $y = 24$ is not awarded • ⁶ .		
10	(a)	$a(t) = -16 \sin\left(2t - \frac{\pi}{2}\right)$	3	
		<ul style="list-style-type: none"> •¹ know to differentiate •² differentiate trig function •³ applies chain rule 		<ul style="list-style-type: none"> •¹ $a = v'(t)$ •² $-8 \sin\left(2t - \frac{\pi}{2}\right) \dots$ •³ $\times 2$ and complete $a(t) = -16 \sin\left(2t - \frac{\pi}{2}\right)$
Notes		1 Alternatively, $8 \cos\left(2t - \frac{\pi}{2}\right) = 8 \sin 2t$ • ¹ $v'(t) \dots$ • ² $= 8 \cos 2t \dots$ • ³ $= \dots \times 2$		

10	(b)	$a(10) > 0$ therefore increasing	2	
		<ul style="list-style-type: none"> •⁴ know to and evaluate $a(10)$ •⁵ interpret result 		<ul style="list-style-type: none"> •⁴ $a(10) = 6 \cdot 53$ •⁵ $a(10) > 0$ therefore increasing
Notes		<p>1 •⁵ is available only as a consequence of substituting into a derivative. 2 •⁴ and •⁵ are not available to candidates who work in degrees. 3 •² and •³ may be awarded if they appear in the working for 10(b). However, •¹ requires a clear link between acceleration and $v'(t)$.</p>		
10	(c)	$s(t) = 4 \sin\left(2t - \frac{\pi}{2}\right) + 8$	3	
		<ul style="list-style-type: none"> •⁶ know to integrate •⁷ integrate correctly •⁸ determine constant and complete 		<ul style="list-style-type: none"> •⁶ $s(t) = \int v(t) dt$ •⁷ $s(t) = 4 \sin\left(2t - \frac{\pi}{2}\right) + c$ •⁸ $c = 8$ so $s(t) = 4 \sin\left(2t - \frac{\pi}{2}\right) + 8$
Notes		<p>4 •⁷ and •⁸ are not available to candidates who work in degrees. However, accept $\int 8 \cos(2t - 90) dt$ for •⁶.</p>		

[END OF EXEMPLAR MARKING INSTRUCTIONS]