H

National
Qualifications
SPECIMEN ONLY

SQ30/H/01

Mathematics
Paper 1
(Non-Calculator)

Marking Instructions

These Marking Instructions have been provided to show how SQA would mark this Specimen Question Paper.

General Marking Principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

(a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) Credit must be assigned in accordance with the specific assessment guidelines.

(d) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(e) Working subsequent to an error must be followed through, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is easier, candidates lose the opportunity to gain credit.

(f) Where transcription errors occur, candidates would normally lose the opportunity to gain a processing mark.

(g) Scored out or erased working which has not been replaced should be marked where still legible. However, if the scored out or erased working has been replaced, only the work which has not been scored out should be judged.

(h) Unless specifically mentioned in the specific assessment guidelines, do not penalise:

- Working subsequent to a correct answer
- Correct working in the wrong part of a question
- Legitimate variations in solutions
- Repeated error within a question

Definitions of Mathematics-specific command words used in this Specimen Question Paper.

Determine: find a numerical value or values from the information given.

Expand: multiply out an algebraic expression by making use of the distributive law or a compound trigonometric expression by making use of one of the addition formulae for $\sin(A \pm B)$ or $\cos(A \pm B)$.

Show that: use mathematics to prove something, eg that a statement or given value is correct — all steps, including the required conclusion, must be shown.

Express: use given information to rewrite an expression in a specified form.

Hence: use the previous answer to proceed.

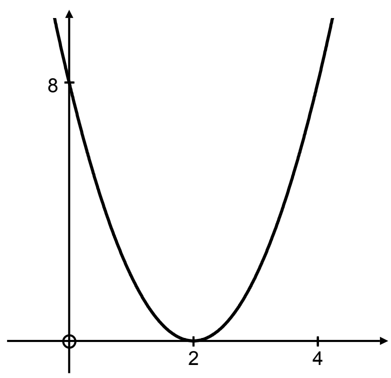
Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used.

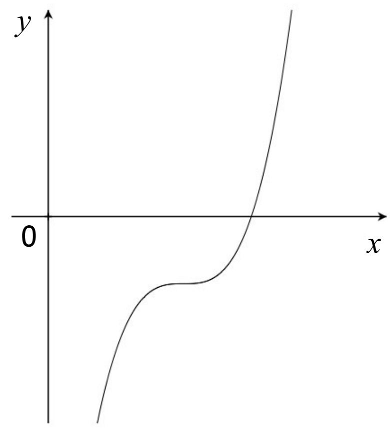
Justify: show good reason(s) for the conclusion(s) reached.

Specific Marking Instructions for each question

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
1			Ans: $\frac{3}{4}x^2 - \frac{1}{2}x^{-1} + C$ •1 preparation for integration •2 correct integration of first term •3 correct integration of second term •4 includes constant of integration	4	•1 $\frac{3}{2}x + \frac{1}{2}x^{-2}$ •2 $\frac{3}{2}\bullet\frac{x^2}{2} + \ldots$ •3 $\ldots + \frac{1}{2}\bullet\frac{x^{-1}}{-1}$ •4 $\frac{3}{4}x^2 - \frac{1}{2}x^{-1} + C$
2			Ans: $(-1,0),(0,4),(3,16)$ •1 sets equation of curve equal to equation of line •2 equates to zero •3 factorises fully •4 calculates x-coordinates •5 calculates y-coordinates	5	•1 $x^3 - 2x^2 + x + 4 = 4x + 4$ •2 $x^3 - 2x^2 - 3x = 0$ •3 $x(x+1)(x-3) = 0$ •4 $x = 0, x = -1, x = 3$ •5 $(0,4),(-1,0),(3,16)$
3			Ans: $S(5,25,-2)$ •1 find coordinate of Q or component vector $\mathbf{q}$ •2 sets up vector equation for $\mathbf{r}$ •3 find coordinate of R or component vector $\mathbf{r}$ •4 sets up vector equation for $\mathbf{s}$ •5 find coordinate of S	5	•1 $\mathbf{q} = \mathbf{p} + \overrightarrow{PQ} = \begin{pmatrix} 0 \\ 15 \\ 3 \end{pmatrix}$ or $Q(0,15,3)$ •2 $\mathbf{r} = \mathbf{q} + \overrightarrow{QR} = \begin{pmatrix} 0 \\ 15 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 6 \\ -3 \end{pmatrix}$ •3 $\mathbf{r} = \begin{pmatrix} 3 \\ 21 \\ 0 \end{pmatrix}$ or $R(3,21,0)$ •4 $\mathbf{s} = \mathbf{r} + \overrightarrow{RS} = \begin{pmatrix} 3 \\ 21 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$ •5 $S(5,25,-2)$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
4			Ans: $-4 < p < 12$ $\bullet^1$ know discriminant < 0 $\bullet^2$ simplify $\bullet^3$ factorise LHS $\bullet^4$ correct range	4	$\bullet^1$ $b^2 - 4ac < 0$ and $a = 2,\ b = p,\ c = p + 6$ stated or implied by $\bullet^2$ $\bullet^2$ $p^2 - 8p - 48 < 0$ $\bullet^3$ $(p-12)(p+4) < 0$ $\bullet^4$ $-4 < p < 12$
5	(a)		Ans: $m_{l_2} = -\sqrt{3}$ $\bullet^1$ rearranging equation to calculate gradient of line l_1 $\bullet^2$ calculating gradient of l_2	2	$\bullet^1$ $y = \frac{1}{\sqrt{3}}x \quad m = \frac{1}{\sqrt{3}}$ $\bullet^2$ $m_{l_2} = -\sqrt{3}$
	(b)		Ans: $\theta = \frac{2\pi}{3}$ or 120° $\bullet^3$ using $m = \tan\theta$ $\bullet^4$ calculating angle	2	$\bullet^3$ $\tan\theta = -\sqrt{3}$ $\bullet^4$ $\theta = \frac{2\pi}{3}$ or 120°
6	(a) (b)		Ans: $\frac{1+\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}+\sqrt{6}}{4}$ $\bullet^1$ **correct** expansion $\bullet^2$ **any** expression equivalent to $\sin 105^\circ$ $\bullet^3$ correct exact value equivalents $\bullet^4$ correct answer	4	$\bullet^1$ $\sin x^\circ \cos 60^\circ + \cos x^\circ \sin 60^\circ$ $\bullet^2$ $\sin(45+60)^\circ$ or equivalent $\bullet^3$ $\frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$ $\bullet^4$ $\frac{1+\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}+\sqrt{6}}{4}$
7	(a)		$\bullet^1$ know to use $x = -1$ $\bullet^2$ complete synthetic division $\bullet^3$ recognition of zero remainder	3	$\bullet^1$ $\begin{array}{r\|rrrr} -1 & 1 & 0 & -13 & -12 \\ & & -1 & 1 & 12 \\ \hline & 1 & -1 & -12 & 0 \end{array}$ $\bullet^2$ $\begin{array}{r\|rrrr} -1 & 1 & 0 & -13 & -12 \\ & & -1 & 1 & 12 \\ \hline & \mathbf{1} & \mathbf{-1} & \mathbf{-12} & \mathbf{0} \end{array}$ $\bullet^3$ $(x+1)$ is a factor as remainder is zero
	(b)		Ans: $(x+1)(x+3)(x-4)$ $\bullet^4$ identify quotient $\bullet^5$ factorised fully	2	$\bullet^4$ $x^2 - x - 12$ $\bullet^5$ $(x+1)(x+3)(x-4)$
Notes			Alternative methods of showing $(x+1)$ is a factor, such as long division, inspection and evaluating are perfectly acceptable.		

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
8	(a)		Ans: $h(x)=2x^2-8x+5$ •[1] correct substitution •[2] squaring •[3] expanding and simplifying	3	•[1] $h(x)=8\left(1-\frac{1}{2}x\right)^2-3$ •[2] $1-x+\frac{1}{4}x^2$ •[3] $h(x)=2x^2-8x+5$
	(b)		Ans: $2(x-2)^2-3$ •[4] identify common factor •[5] complete the square •[6] process for q	3	•[4] $2(x^2-4x\ldots$ **stated or implied by •[3]** •[5] $2(x-2)^2\ldots$ •[6] $2(x-2)^2-3$
Notes			Values for p and q must be consistent with the value for a.		
	(c)		Ans: $(2,-3)$ •[7] state turning point	1	•[7] $(2,-3)$
	(d)		Ans:  •[8] correct shape •[9] annotation, including y-axis intercept	2	•[8] parabola with minimum turning point labelled (positioned consistently with answer to (b)) •[9] (0,8)

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
9	(a)		Ans: $y-10=-3(x+1)$ •[1] finding equation of line	1	 •[1] $y-10=-3(x+1)$ or equivalent
	(b)		Ans: $B(3,-2)$ •[2] use of simultaneous equations •[3] solving to find one coordinate of midpoint •[4] finding remaining coordinate of midpoint •[5] using midpoint formula or 'stepping out' •[6] finding coordinates of B	5	•[2] $y=-3x+7$ and $3y=x+11$ •[3] either $x=1$ or $y=4$ •[4] M (1, 4) •[5] either $x=3$ or $y=-2$ •[6] $B(3,-2)$
10			Ans: $\frac{3\sqrt{3}}{2}$ •[1] start to differentiate •[2] complete differentiation •[3] evaluate $f'\left(\frac{5\pi}{6}\right)$	3	•[1] $3\times4\sin^2 x$ •[2] $\times\cos x$ •[3] $12\left(\frac{1}{2}\right)^2\times\frac{-\sqrt{3}}{2}=12\times\frac{1}{4}\times\frac{-\sqrt{3}}{2}=\frac{-3\sqrt{3}}{2}$
11	(a)		•[1] knows derived function represents gradient and that the minimum value of $f'(x)$ is zero	1	•[1] $m=f'(x)\geq0$ stated explicitly
	(b)		•[2] interprets information correctly •[3] completes sketch	2	•[2] stationary point plotted in fourth quadrant •[3] point of inflexion on an increasing graph 

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
12	(a)		Ans: $\frac{1}{50}$ sec or 0·02 sec	**2**	
			•[1] knows how to find period		•[1] $T=\frac{2\pi}{100\pi}$
			•[2] correct answer		•[2] $\frac{1}{50}$ or 0·02
	(b)		Ans: $\frac{7}{600}$, $\frac{11}{600}$, and $\frac{19}{600}$ sec	**6**	
			•[1] equating function with –60		•[1] $120\sin 100\pi t=-60$
			•[2] rearranging		•[2] $\sin 100\pi t=-\frac{1}{2}$
			•[3] solve equation for $100\pi t$		•[3] $100\pi t=\frac{7\pi}{6}$ and $\frac{11\pi}{6}$ (•[3])
			•[4] process solutions for t		•[4] $t=\frac{7}{600}$ and $\frac{11}{600}$ (•[4])
			•[5] knowing to use period or demonstrating another solution from the third quadrant		•[5] $T=\frac{1}{50}$ or $100\pi t=3\pi+\frac{\pi}{6}$
			•[6] third value for t		•[6] $\frac{19}{600}$

[END OF SPECIMEN MARKING INSTRUCTIONS]

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Mathematics
Paper 2

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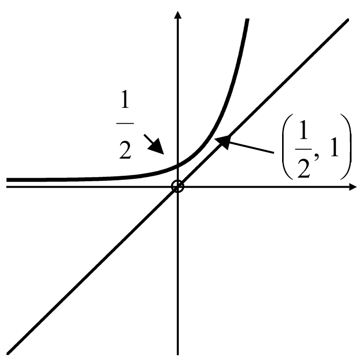
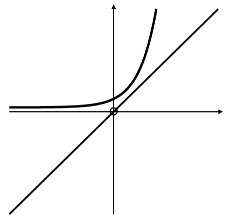
1	(a)		•[1] find coordinates of E •[2] find coordinates of F	2	•[1] E(45, 45, 40) •[2] F(60, 40, 0)
	(b)		Ans: -1750 •[3] find $\overrightarrow{ED}$ and $\overrightarrow{EF}$ •[4] correct calculation of scalar product	2	•[3] $\overrightarrow{ED}=\begin{pmatrix}-15\\-15\\40\end{pmatrix}$, $\overrightarrow{EF}=\begin{pmatrix}15\\-5\\-40\end{pmatrix}$ •[4] $\overrightarrow{ED}.\overrightarrow{EF}=-225+75-1600=-1750$
	(c)		Ans: 154° •[5] know how to find angle DEF using formula •[6] find $\lvert\overrightarrow{ED}\rvert$ •[7] find $\lvert\overrightarrow{EF}\rvert$ •[8] calculates angle DEF	4	•[5] $\cos DEF=\dfrac{\overrightarrow{ED}.\overrightarrow{EF}}{\lvert\overrightarrow{ED}\rvert\lvert\overrightarrow{EF}\rvert}$ or equivalent •[6] $\lvert\overrightarrow{ED}\rvert=\sqrt{2050}$ •[7] $\lvert\overrightarrow{EF}\rvert=\sqrt{1850}$ •[8] $\cos DEF=\dfrac{-1750}{\sqrt{2050}\sqrt{1850}}$ $DEF=153\cdot977...=154°$
2	(a)		Ans: $a=0\cdot96$, $b=580$ •[1] set up one equation •[2] set up second equation •[3] solve for one variable •[4] solve for second variable	4	•[1] $2500=2000a+b$ •[2] $2980=2500a+b$ •[3] $480=500a$ or $12500=10000a+5b$ $a=\dfrac{480}{500}$ $\quad 11920=10000a+4b$ $a=0\cdot96$ $\quad 580=b$ •[4] $b=2500-2000(0\cdot96)$ $b=2500-1920$ $b=580$ or $2000a=2500-580$ $a=\dfrac{1920}{2000}$ $a=0\cdot96$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
	(b)		Ans: Yes. Stabilises at 14500	3	
			•[5] knows how to find the limit		•[5] $u_{n+1} = 0\cdot96u_n + 580, \quad -1 < a < 1$ $L = \frac{b}{1-a}$ $L = \frac{580}{1-0\cdot96}$
			•[6] calculate limit		•[6] $L = 14500$
			•[7] conclusion		•[7] yes, conservation measures will end, since the predicted population stabilises at 14500 and $14500 > 13000$
3	(a)		Ans: $p = 1, q = 4$	1	
			•[1] state values of p and q		•[1] $p = 1, q = 4$
	(b)		Ans: $y = 9(x-1)$	4	
			•[2] expand brackets		•[2] $f(x) = x^4 - 9x^3 + 24x^2 - 16x$
			•[3] differentiate		•[3] $f'(x) = 4x^3 - 27x^2 + 48x - 16$
			•[4] calculate gradient of tangent		•[4] $f'(1) = 4 - 27 + 48 - 16 = 9$
			•[5] substitutes gradient and (1,0) into equation of line		•[5] $y = 9(x-1)$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
4	(a)		Ans: $y = \log_4 x + \frac{1}{2}$ •[1] using law of logarithms •[2] evaluating $\log_4 2$	2	•[1] $\log_4 2x = \log_4 2 + \log_4 x$ •[2] $\log_4 2 = \frac{1}{2}$
	(b)		Ans: Graph of $y = \log_4 x$ moved up by $\frac{1}{2}$ or graph of $y = \log_4 x$ compressed horizontally by a factor of 2. •[3] valid description of relationship	1	•[3] valid description — see answer
	(c)		Ans: $x = \frac{1}{2}$ •[4] setting $y = 0$ •[5] solving for x	2	•[4] $\log_4 2x = 0$ •[5] $x = \frac{1}{2}$
	(d)		Ans:  •[6] reflecting $y = \log_4 2x$ in the line $y = x$ •[7] correct shape •[8] annotating (2 points) **(or other valid method)**	3	•[6] reflect in $y = x$ •[7]  •[8] $\left(0, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, 1\right)$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
5	(a)		Ans: $y-1=-2(x-3)$	4	
			•[1] calculate midpoint of AB		•[1] (3, 1)
			•[2] calculate gradient of AB		•[2] $\frac{1}{2}$
			•[3] state gradient of perpendicular bisector		•[3] -2
			•[4] substitute into equation of line		•[4] $y-1=-2(x-3)$
	(b)		Ans: $(x-2)^2+(y-3)^2=25$	4	
			•[5] knowing and using $y=3$		•[5] $3=-2x+7$
			•[6] solving for x		•[6] $x=2$
			•[7] identifying the radius		•[7] $r=5$
			•[8] obtain circle equation		•[8] $(x-2)^2+(y-3)^2=25$
6			Ans: $x=-3$	5	
			•[1] use perpendicular property		•[1] If $\overrightarrow{CD}$ is perpendicular to $\overrightarrow{AB}$ then $\overrightarrow{CD}.\overrightarrow{AB}=0$
			•[2] find $\overrightarrow{CD}$		•[2] $\begin{pmatrix} x-4 \\ -3 \\ -1 \end{pmatrix}$
			•[3] find $\overrightarrow{AB}$		•[3] $\begin{pmatrix} 5 \\ -10 \\ -5 \end{pmatrix}$
			•[4] correct substitution into scalar product formula		•[4] $5(x-4)+(-10)(-3)+(-5)(-1)=0$
			•[5] calculates value of x		•[5] $x=-3$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
7			Ans: A False and B True •[1] valid reason for statement A •[2] selecting true or false for statement A with valid reason •[3] setting $P(t)=15$ •[4] taking log to base e •[5] completing valid reason •[6] selecting true or false for statement B with valid reason	6	•[1] $P(0)=30e^{-2}=4\cdot06$ •[2] false, since $P(0)\neq30$ (do not award without valid reason) •[3] $15=30e^{t-2}$ •[4] $\ln e^{t-2}=\ln0\cdot5$ •[5] $t-2=\ln0\cdot5$ $t=\ln0\cdot5+2$ (1·3) •[6] true, since $t=1\cdot3$ to one decimal place and there is only one solution (do not award without valid reason)
Notes			Substituting $t=1\cdot3$ into $P(t)=30e^{t-2}$ is not sufficient to show that statement B is true, since it does not prove that $t=1\cdot3$ is the <u>only</u> solution.		
8	(a)		•[1] know to equate volume to 100 •[2] obtain an expression for h •[3] complete area evaluation	3	•[1] $V=\pi r^2h=100$ •[2] $h=\dfrac{100}{\pi r^2}$ •[3] $A=\pi r^2+2\pi r^2+2\pi r\times\dfrac{100}{\pi r^2}$
	(b)		Ans: $r=2\cdot20$ m •[4] know to and start to differentiate •[5] complete differentiation •[6] set derivative to zero •[7] obtain r •[8] justify nature of stationary point •[9] interpret result	6	•[4] $A'(r)=6\pi r...$ •[5] $A'(r)=6\pi r-\dfrac{200}{r^2}$ •[6] $6\pi r-\dfrac{200}{r^2}=0$ •[7] $r=2\cdot20$ metres •[8] $A''(r)=6\pi+\dfrac{400}{r^3}\Rightarrow A''(2\cdot1974...)=56\cdot5...$ •[9] minimum (when $r=2\cdot20$ m)
Notes			Candidates may use a nature table at •[8] to justify a minimum turning point when $r=2\cdot1974...$		

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
9			Ans: $\frac{5}{6}$ •1 knowing to use integration •2 using correct limits •3 integrating correctly •4 integrating correctly •5 substituting limits correctly •6 evaluating correctly	6	 •1 $\int \sin(\frac{3}{4}x - \frac{3}{2}\pi)dx - \int \cos(2x)dx$ •2 $\int_0^{\frac{2}{3}\pi} \sin(\frac{3}{4}x - \frac{3}{2}\pi)dx - \int_0^{\frac{\pi}{4}} \cos(2x)dx$ •3 $\left[-\frac{4}{3}\cos(\frac{3}{4}x - \frac{3}{2}\pi)\right]$……. •4 $-[\frac{1}{2}\sin(2x)]$ •5 See * below •6 $(\frac{4}{3} - 0) - (\frac{1}{2} - 0) = \frac{5}{6}$
* $([-\frac{4}{3}\cos(\frac{3}{4}\times\frac{2}{3}\pi - \frac{3}{2}\pi)] - [-\frac{4}{3}\cos(0 - \frac{3}{2}\pi)]) - ([\frac{1}{2}\sin(2\times\frac{1}{4}\pi)] - [\frac{1}{2}\sin(2\times 0)])$					
10	(a)		Ans: $k = 2, \alpha = \frac{\pi}{6}$ or equivalent •1 knows to set wave function equal to addition of individual waves •2 knows to expand •3 knows to compare coefficients •4 interpret comparison	4	 •1 $\sin t + \sqrt{3}\cos t = k\cos(t - \alpha)$ or equivalent •2 $k\cos\alpha\cos t + k\sin\alpha\sin t$ or equivalent •3 $k\sin\alpha = 1, \quad k\cos\alpha = \sqrt{3}$ or equivalent •4 $k = 2, \quad \alpha = \frac{\pi}{6}$ or equivalent
			Ans: 5·9 •5 equates wave function with y-coordinate of P •6 rearranges correctly •7 solve equation for $t - \frac{\pi}{6}$ •8 find t-coordinate of P by interpreting diagram	4	•5 $2\cos\left(t - \frac{\pi}{6}\right) = 1\cdot 2$ or equivalent •6 $\cos\left(t - \frac{\pi}{6}\right) = 0\cdot 6$ or equivalent •7 $t - \frac{\pi}{6} = 0\cdot 927\ldots$ & $5\cdot 355\ldots$ $1\cdot 45\ldots$ & •8 $5\cdot 879\ldots$

[END OF SPECIMEN MARKING INSTRUCTIONS]